

HIGH SCHOOL CORE LEARNING GOALS

ALGEBRA/DATA ANALYSIS

Core Learning Goals for:

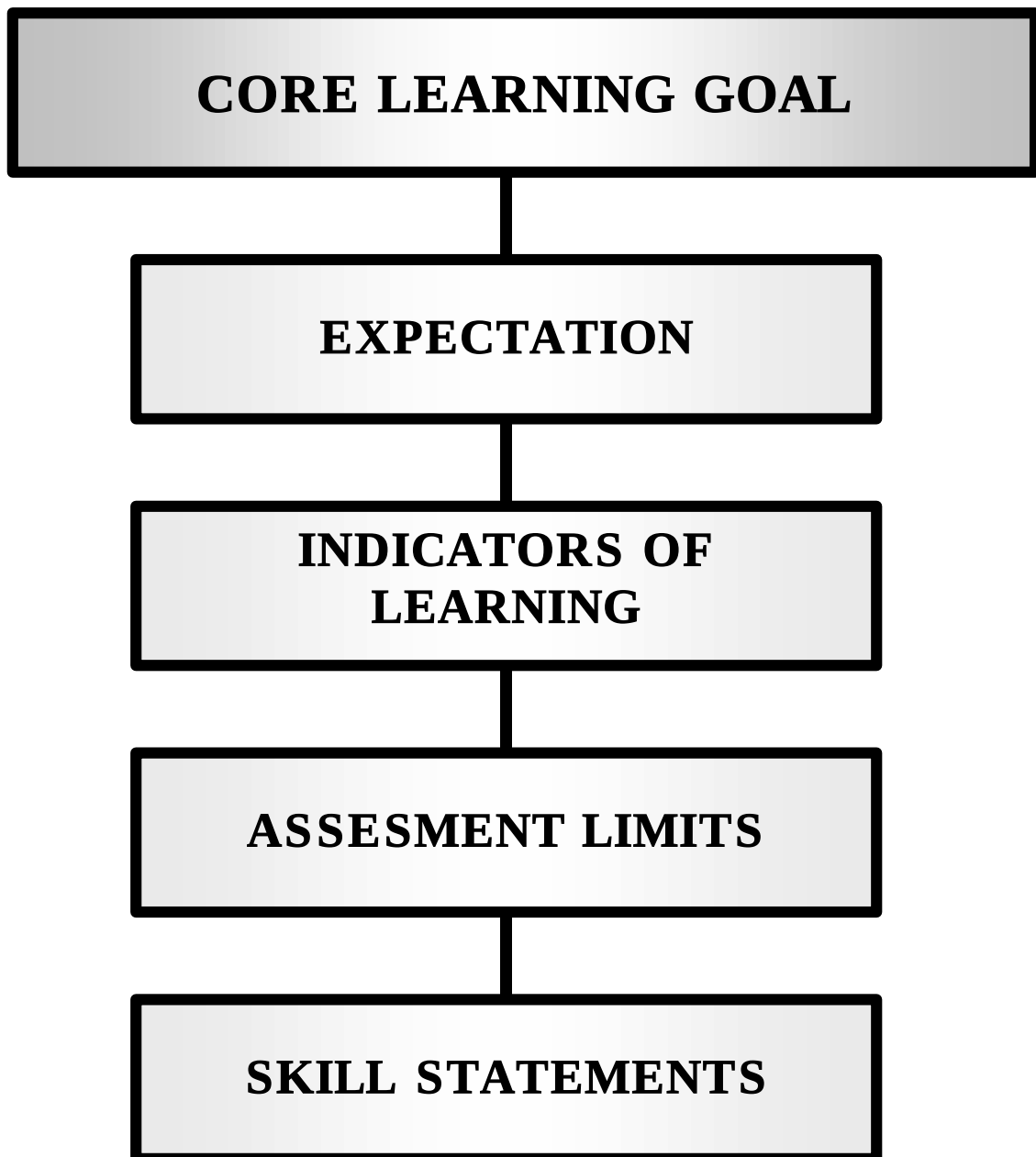
- Functions and Algebra
- Geometry Measurements and Reasoning
- Data Analysis and Probability

Updated March 2001



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HIGH SCHOOL ASSESSMENT CORE LEARNING GOALS FORMAT MATHEMATICS



MATHEMATICS CORE LEARNING GOALS

GOAL 1: FUNCTIONS AND ALGEBRA

The student will demonstrate the ability to investigate, interpret, and communicate solutions to mathematical and real-world problems using patterns, functions, and algebra.

GOAL 2: GEOMETRY, MEASUREMENT, AND REASONING

The student will demonstrate the ability to solve mathematical and real-world problems using measurement and geometric models and will justify solutions and explain processes used.

GOAL 3: DATA ANALYSIS AND PROBABILITY

The student will demonstrate the ability to apply probability and statistical methods for representing and interpreting data and communicating results, using technology when needed.

Comments

The following comments are applicable to every expectation:

- The use of “ and,” “and/or”, and “or” should be interpreted for instructional purposes to mean that all of the mathematics included in a goal, expectation, or indicator should be part of the instruction. For assessment purposes, however, it is unlikely that an item will include all of the mathematics included in a goal, expectation, or indicator.
- Instruction at all levels should include opportunities for students to read, interpret, and apply mathematics in context from textbooks and appropriate materials in a variety of ways. Assignments that require students to read mathematics and respond both orally and in writing to questions based on their reading should be an integral part of the mathematics program.
- Multiple approaches to attain the indicators are encouraged.
- Units of measurement can be metric or customary.
- In the Goal 2 statement:
 - ‘Justify conclusions’ means the student will use mathematical principles to support the reasoning used to solve the problem or to demonstrate that the solution is correct. This could include the appropriate definitions, postulates, and theorems.
 - ‘Explain the processes used’ means the student will use the language of mathematics to communicate how the student arrived at the solution.

GOAL 1: FUNCTIONS AND ALGEBRA

The student will demonstrate the ability to investigate, interpret, and communicate solutions to mathematical and real-world problems using patterns, functions, and algebra.

1.1 ***Expectation: The student will analyze a wide variety of patterns and functional relationships using the language of mathematics and appropriate technology.***

Indicators

- 1.1.1 The student will recognize, describe, and/or extend patterns and functional relationships that are expressed numerically, algebraically, and/or geometrically.

Assessments Limits

- The given pattern must represent a relationship of the form $y = mx + b$ (linear), $y = x^2 + c$ (simple quadratic), $y = x^3 + c$ (simple cubic), simple arithmetic progression, or simple geometric progression with all exponents being positive.
- The student will not be asked to draw three- dimensional figures.
- Algebraic description of patterns is in indicator 1.1.2

Skill Statement

- Given a narrative, numeric, algebraic, or geometric representation description of a pattern or functional relationship, the student will give a verbal description, or predict the next term or a specific term in a pattern or functional relationship.
 - Given a numerical or graphical representation of a relation, the student will identify if the relation is a function and/or describe it.
- 1.1.2 The student will represent patterns and/or functional relationships in a table, as a graph, and/or by mathematical expression.

Assessment Limits

- The given pattern must represent a relationship of the form $mx + b$ (linear), x^2 (simple quadratic), simple arithmetic progression, or simple geometric progression with all exponents being positive.

Skill Statement

Given a narrative description, algebraic expression, graph or table, the student will produce a graph, table, algebraic expression of the form $mx + b$ (linear) or x^2 (simple quadratic), or equation.

- 1.1.3 The student will apply addition, subtraction, multiplication, and/or division of algebraic expressions to mathematical and real-world problems.

Assessment Limit

- The algebraic expression is a polynomial in one variable.
- The polynomial is not simplified.

Skill Statement

The student will represent a situation as a sum, difference, product, and/or quotient in one variable.

- 1.1.4 The student will describe the graph of a non-linear function and discuss its appearance in terms of the basic concepts of maxima and minima, zeros (roots), rate of change, domain and range, and continuity.

Assessment Limits

- A coordinate graph will be given with easily read coordinates.
- “Zeros” refers to the x-intercepts of a graph, “roots” refers to the solution of an equation in the form $p(x) = 0$.
- Problems will not involve a real-world context.

Skill Statement

Given the graph of a non-linear function, the student will identify maxima/minima, zeros, rate of change over a given interval (increasing/decreasing), domain and range, or continuity.

1.2 *Expectation: The student will model and interpret real-world situations using the language of mathematics and appropriate technology.*

Indicators

- 1.2.1 The student will determine the equation for a line, solve linear equations, and/or describe the solutions using numbers, symbols, and/or graphs.

Assessment Limits

- Functions are to have no more than two variables with rational coefficients.
- Linear equations will be given in the form:
 $Ax + By = C$, $Ax + By + C = 0$, or $y = mx + b$.
- Vertical lines are included.
- The majority of these items should be in real-world context.

Skill Statement

Given one or more of the following:

- the graph of a line
- written description of a situation that can be modeled by a linear function
- two or more collinear points
- a point and slope

the student will do one or more of the following:

- write the equation
- solve a one-variable equation for the unknown

- solve a two-variable equation for one of the variables
- graph the resulting equation
- interpret the solution in light of the context
- evaluate the equation for a given value
- create a table of values
- find and/or interpret the slope (rate of change) and/or intercepts in relation to the context.

Any correct form of a linear equation will be acceptable as a response.

- 1.2.2 The student will solve linear inequalities and describe the solutions using numbers, symbols, and/or graphs.

Assessment Limits

- Inequalities will have no more than two variables with rational coefficients.
- Acceptable forms of the problem or solution are the following:
 $Ax + By < C$, $Ax + By \leq C$, $Ax + By > C$, $Ax + By \geq C$, $Ax + By + C < 0$,
 $Ax + By + C \leq 0$, $Ax + By + C > 0$, $Ax + By + C \geq 0$, $y < mx + b$,
 $y \leq mx + b$, $y \geq mx + b$, $y > mx + b$, $y < b$, $y \leq b$, $y > b$, $y \geq b$, $x < b$,
 $x \leq b$, $x > b$, $x \geq b$, $a \leq x \leq b$, $a < x < b$, $a \leq x < b$, $a < x \leq b$,
 $a \leq x + c \leq b$, $a < x + c < b$, $a \leq x + c < b$, $a < x + c \leq b$.
- The majority of these items should be in real-world context.
- Systems of linear inequalities will not be included.
- Compound inequalities will be included.
- Disjoint inequalities will not be included.
- Absolute value inequalities will not be included.

Skill Statement

- Given a linear inequality in narrative, algebraic, or graphical form, the student will graph the inequality, write an inequality and/or solve it, or interpret an inequality in the context of the problem.
- Any correct form of a linear inequality will be an acceptable response.

- 1.2.3 The student will solve and describe using numbers, symbols, and/or graphs if and where two straight lines intersect.

Assessment Limits

- Functions will be of the form: $Ax + By = C$, $Ax + By + C = 0$, or $y = mx + b$.
- All coefficients will be rational.
- Vertical lines will be included.
- Systems of linear functions will include coincident, parallel, or intersecting lines.
- The majority of these items should be in real-world context.

Skill Statement

Given one or more of the following:

- a narrative description
- the graph of two lines
- equations for two lines

the student will do one or more of the following:

- determine the system of equations and/or its solution
- describe the relationship of the points on one line with points on the other line

- give the meaning of the point of intersection in the context of the problem
 - graph the system, determine the solution and interpret the solution in the context of the problem
 - use slope to recognize the relationship between parallel lines.
- Any correct form of a linear equation will be an acceptable response.

1.2.4 The student will describe how the graphical model of a non-linear function represents a given problem and will estimate the solution.

Assessment Limits

- The problem is to be in a real-world context.
- The function will be represented by a graph.
- The equation of the function may be given.
- The features of the graph may include maxima/minima, zeros (roots), rate of change over a given interval (increasing/decreasing), continuity, or domain and range.
- “Zeros” refers to the x-intercepts of a graph, “roots” refers to the solution of an equation in the form $p(x) = 0$.
- Functions may include step, absolute value, or piece-wise functions.

Skill Statement

Given a graph which represents a real-world situation, the student will describe the graph and/or explain how the graph represents the problem or solution and/or estimate a solution.

1.2.5 The student will apply formulas and/or use matrices (arrays of numbers) to solve real-world problems.

Assessment Limits

- Formulas will be provided in the problem or on the reference sheet.
- Formulas may express linear or non-linear relationships.
- The students will be expected to solve for first degree variables only.
- Matrices will represent data in tables.
- Matrix addition, subtraction, and/or scalar multiplication may be necessary.
- Inverse and determinants of matrices will not be required.

Skill Statement

- Given a formula, students will substitute values, solve and interpret solutions in the context of a problem.
- Given matrices, the students will perform operations and interpret solutions in real-world contexts.

GOAL 2: GEOMETRY, MEASUREMENT, AND REASONING

The student will demonstrate the ability to solve mathematical and real-world problems using measurement and geometric models and will justify solutions and explain process used.

2.1 *Expectation: The student will represent and analyze two-and three-dimensional figures using tools and technology when appropriate.*

Indicators

2.1.1 The student will analyze the properties of geometric figures.

Assessment Limits

Essential properties, relationships, and geometric models include the following:

- Congruence and similarity
- line/segment/plane relationships (parallel, perpendicular, intersecting, bisecting, midpoint, median, altitude)
- point relationships (collinear, coplanar)
- angles and angle relationships (vertical, adjacent, complementary, supplementary, obtuse, acute, right, interior, exterior)
- angle relationships with parallel lines
- polygons (regular, non-regular, composite, equilateral, equiangular)
- geometric solids (cones, cylinders, prisms, pyramids, composite figures)
- circle/sphere (tangent, radius, diameter, chord, secant, central/inscribed angle, inscribed, circumscribed).

Skill Statement

The student describes and analyzes geometric figures.

2.1.2 The student will identify and/or verify properties of geometric figures using the coordinate plane and concepts from algebra.

Assessment Limits

- “Verify properties” means to justify solutions using definitions and/or mathematical principles.
- Properties, relationships, and geometric models include the following:
 - Congruence and similarity
 - line/segment relationships (parallel, perpendicular, intersecting, bisecting, midpoint, median, altitude)
 - point relationships (collinear)
 - angles and angle relationships (obtuse, acute, right)
 - polygons (regular, non-regular, equilateral, equiangular)
 - circle (tangent, radius, diameter, chord).
- Items for this indicator may be set on the coordinate plane or may just have coordinates identified with no grid.
- Concepts from algebra include applications of the distance, midpoint, and slope formulas.

Skill Statement

The student uses the coordinate plane and algebra to analyze geometric figures.

- 2.1.3 The student will use transformations to move figures, create designs, and/or demonstrate geometric properties.

Assessment Limits

- Transformations include reflections, rotations, translations, and dilations.
- Items should go beyond the identification of transformations.
- Essential properties and relationships include the following: congruence, similarity, and symmetry.
- The student's explanation of a transformation must include the following:
 - translation – distance and direction
 - reflection – line of reflection
 - rotation – center of rotation, angle measure, direction (clockwise or counterclockwise)
 - dilation – center and scale factor
- Paper folding and the use of Miras™ and mirrors are appropriate methods for performing transformations, and their use must be referenced.

Skill Statement

Given one or more transformations, the student sketches the result of the transformation(s) and/or explains the geometric effect of the transformation(s) on the figure.

- 2.1.4 The student will construct and/or draw and/or validate properties of geometric figures using appropriate tools and technology.

Assessment Limits

- “Validate properties” in this indicator, means justifying solutions using definitions, mathematical principles and/or measurement.
- Students may use a compass, straightedge, patty paper, a Mira™, and/or a mirror as construction tools. Using a ruler or protractor cannot be part of the strategy.
- Students may use a compass, ruler, patty paper, a Mira™, a mirror and/or a protractor as drawing tools.
- It is acceptable to do a construction when the item asks for a drawing.
- Paper folding and the use of Miras™ and mirrors are appropriate methods for representing, constructing, and/or analyzing figures, and their use must be referenced.
- Constructions and drawings are limited to the two-dimensional relationships listed in 2.1.1.

Skill Statement

The student draws and/or constructs geometric figures and/or justifies the solution.

2.2 Expectation: The student will apply geometric properties and relationships to solve problems using tools and technology when appropriate.

Indicators

- 2.2.1 The student will identify and/or verify congruent and similar figures and/or apply equality or proportionality of their corresponding parts.

Assessment Limits

- Students will demonstrate geometric reasoning and justify conclusions. Although the focus is on geometric theory, answers to some items may include a numeric answer.
- Corresponding measurements include length, angle measure, perimeter, circumference, area, volume, surface area and lateral area.

Skill Statement

- The student recognizes shape as congruent or similar, calculates corresponding measurements, and/or justifies conclusions.
- The student uses congruency and similarity statements to identify corresponding parts of figures.

- 2.2.2 The student will solve problems using two-dimensional figures and/or right-triangle trigonometry.

Assessment Limits

- Students will demonstrate geometric reasoning and justify conclusions.
- Trigonometric functions may be used to find sides or angles.
- Trigonometric functions will be limited to sine, cosine, and tangent and their inverses.

Skill Statement

The student solves a problem involving missing parts of two-dimensional figures, which may require the use of right-triangle trigonometry, the Pythagorean theorem, or special right triangle relationships.

- 2.2.3. The student will use inductive or deductive reasoning.

Assessment Limits

- Students are expected to demonstrate their geometric reasoning and justify conclusions. Although the focus is on geometric theory, answers to some questions may include a numeric answer.
- Items may include geometric applications, patterns, and logic, including syllogisms.
- Narrative, flow chart, or two-column proof may be used as a valid argument.

Skill Statement

Given a situation, the student arrives at or justifies a conclusion using inductive or deductive reasoning.

2.3 Expectation: The student will apply concepts of measurement using tools and technology when appropriate.

Indicators

2.3.1. The student will use algebraic and/or geometric properties to measure indirectly.

Assessment Limits

- “Measure indirectly” means to use mathematical concepts such as congruence, similarity, and ratio and proportion to calculate measurements.
- Similarity and congruence will be directly stated or implied (scale drawings, enlargements).
- Items may require the student to make comparisons.
- This indicator may incorporate measuring.
- This indicator does not include right-triangle trigonometry.

Skill Statement

The student calculates measurements indirectly by using mathematical concepts.

2.3.2 The student will use techniques of measurement and will estimate, calculate, and/or compare perimeter, circumference, area, volume, and/or surface area of two-and three-dimensional figures and their parts.

Assessment Limits

- Two-dimensional shapes include polygons, circles, and composite figures.
- Three-dimensional shapes include cubes, prisms, pyramids, cylinders, cones, spheres, and composite figures.
- Formulas will be provided.
- No oblique solids will be used.
- Items may involve applications of geometric properties and relationships.
- Students may be required to make comparisons which do not require calculations.

Skill Statement

The student solves a problem involving perimeter, area, surface area, lateral area, circumference, and/or volume expressing solutions with appropriate units.

GOAL 3: DATA ANALYSIS AND PROBABILITY

The student will demonstrate the ability to apply probability and statistical methods for representing and interpreting data and communicating results, using technology with needed.

3.1 Expectation: The student will collect, organize, analyze, and present data.

Indicators:

- 3.1.1 The student will design and/or conduct an investigation that uses statistical methods to analyze data and communicate results.

Assessment Limits

- The student will design investigations stating how data will be collected and justify the method.
- Types of investigations may include: simple random sampling, representative sampling, and probability simulations.
- Probability simulations may include the use of spinners, number cubes, or random number generators.
- In simple random sampling each member of the population is equally likely to be chosen and the members of the sample are chosen independently of each other. Sample size will be given for these investigations.

Skill Statement

- The student will design an investigation and justify their design.
- The students will describe how they would do an investigation, select a sampling technique and justify their choice
- The student will demonstrate an understanding of the concepts of bias, sample size, randomness, representative samples, and simple random sampling techniques.

- 3.1.2 The student will use the measures of central tendency and/or variability to make informed conclusions.

Assessment Limits:

- Measures of central tendency include mean, median, and mode.
- Measures of variability include range, interquartile range, and quartiles.
- Data may be displayed in a variety of representations which may include: frequency tables, box and whisker plots, and other displays.

Skill Statement

- The student uses measures of central tendency and variability to solve problems, make informed conclusions and/or display data.
- The student will recognize and apply the effect of the distribution of the data on the measures of central tendency and variability.

- 3.1.3 The student will calculate theoretical probability or use simulations or statistical inference from data to estimate the probability of an event.

Assessment Limits

- This indicator does not include finding probabilities of dependent events.

Skill Statement

- Using given data, the student determines the experimental probability of an event.
- Given a situation involving chance, the student will determine the theoretical probability of an event.

3.2 *Expectation: The student will apply the basic concepts of statistics and probability to predict possible outcomes of real-world situations.*

Indicator

- 3.2.1 The student will make informed decisions and predictions based upon the results of simulations and data from research.

Skill Statement

Given data from a simulation or research, the student makes informed decisions and predictions.

- 3.2.2 The student will interpret data and/or make predictions by finding and using a line of best fit and by using a given curve of best fit.

Assessment Limits

- Items should include a definition of the data and what it represents.
- Data will be given when a line of best fit is required.
- Equation or graph will be given when a curve of best fit is required.

Skill Statement

- The students will find a line of best fit, use it to interpolate and extrapolate, and/or interpret slope and intercepts.
- The student will use a curve of best fit to interpolate and extrapolate.
- The student's response will be in the context of the problem.

- 3.2.3 The student will communicate the use and misuse of statistics.

Assessment Limits

- Examples of "misuse of statistics" include the following:
 - misuse of scaling on a graph
 - misuse of measures of central tendency and variability to represent data,
 - using three-dimensional figures inappropriately
 - using data to sway interpretation to a predetermined conclusion
 - using incorrect sampling techniques
 - using data from simulations incorrectly
 - predicting well beyond the data set.

Skill Statement

The student will analyze and identify proper and improper use of statistical data and/or statistical methods.